Dynamic Imaging and Fast Reconstruction Algorithms in Tomography

A Final Technical Report For the JSEP Graduate Fellowship

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Dear Sirs:

Enclosed is the final report and transcript for Samit Basu for work he has done while supported on a JSEP Fellowship during the 1998-1999 academic year. Samit will be completing his Ph.D. this year.

If you need any further information, please feel free to contact me.

Sincerely,

N. Narayana Rao

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Abstract

During the period of sponsorship under the JSEP fellowship, the researcher has studied tomographic imaging, the mathematical process that underlies a number of technologies such as Synthetic Aperture Radar, X-ray computed tomography, Magnetic Resonance Imaging, etc. In particular, his research has focused on two key aspects of tomography. The first is the study of tomography in the presence of motion. In this area, he was the first to establish conditions for unique and stable reconstruction in the presence of rigid body motion. The second aspect is that of fast algorithms for tomography. He has developed new advanced algorithms that permit tomographic imaging to be carried out much faster than was previously possible. In the process of studying these two key areas, he has developed new and fundamental results in signal processing.

Contents

Chapter 1

Introduction

Goals:

Our research work focused on two goals. Our first goal was to study tomographic reconstruction of time-varying densities by means of motion models. Our second goal was to study and develop a new class of fast algorithms for tomography.

Motivation:

Tomography is a technique by which a function defined in the plane is reconstructed from a collection of line integral measurements at different orientations [1]. It has found numerous applications in the fields of medical imaging, nondestructive evaluation, radio astronomy, Synthetic Aperture Radar (SAR), electron microscopy based tomography, and nondestructive evaluation (NDE) among others. As such, it is a pervasive technique and has been well studied.

The mathematical characterization of the tomographic problem was initially described by Radon in 1917 [2]. Radon constructed an inversion formula for tomographic data in 2D, when the line integrals were available from all possible directions. With the introduction of the Computed Tomography (CT) scanner, it became possible to collect these line integral measurements, albeit at a finite set of directions. Much work over the last few decades has focused on reconstruction techniques from such finite sets of data [3]. Many techniques have been described and implemented, and tomographic reconstruction is generally considered a solved problem.

However, as imaging technology improves, the limitations of the traditional tomographic formulation are beginning to manifest themselves [4]. We have studied two of these limitations in this work. The first is the limitation of assuming a temporally-static 2-D density. In most practical situations, the acquisition is time-sequential, so that different sets of measurements are taken at sequential moments in time. If the 2-D density being imaged is evolving in time, then the measurements violate the assumption that each measurement is of the same 2-D density. This violation leads to imaging artifacts that can render the reconstructions useless. Indeed, extensive documentation exists of the types of artifacts that occur in medical imaging scenarios as a result of voluntary (movement of the arms, head, etc.) and involuntary patient motion (cardiac, gastrointestinal, etc), and the resulting effect on the diagnostic utility of the reconstruction [5, 6]. The second limitation we have examined is that of computational complexity. Traditional techniques for reconstruction are generally computationally demanding. As real-time reconstruction becomes increasingly important. the push for faster reconstruction algorithms with little distortion becomes more significant as well [4]. With the recent ground-breaking introduction of a hierarchical reprojection algorithm in [7]. it may be possible to achieve this goal.

Organization:

In Chapter 2, we review the state of the art on the issues of tomography in the presence of motion and on fast algorithms for tomography. In Chapter 3, we describe the results achieved while sponsored by the JSEP fellowship. This includes fundamental, new results in tomographic reconstruction in the presence of motion, as well as a new class of fast algorithms for tomography. Some fundamental results in signal processing that we developed as a consequence of our primary research interests are also described there. In Chapter 4, we discuss our conclusions regarding the work we have completed. We have attached a list of submitted and published papers written during the duration of this fellowship. In accordance with the requirements for this final report, these papers are *not* attached as appendices.

Chapter 2

Review

I. Motion in Tomography

A. Formulation

As traditionally formulated, the tomographic reconstruction problem is to reconstruct an image supported on the unit disk $\mathbb{B}_2$, from a set of P parallel beam projections at view angles $\theta_i \in [0, \pi)$. If we assume that the image belongs to the Hilbert space of square-integrable functions supported on $\mathbb{B}_2$ (denoted $L_2(\mathbb{B}_2)$), then we can define a Radon transform operator $\mathcal{R} : L_2(\mathbb{B}_2) \mapsto L_2^P(\mathbb{B}_1)$ as

$$\mathcal{R}f(s,i) = \int_{-\sqrt{1-s^2}}^{\sqrt{1-s^2}} f(s\cos\theta_i + r\sin\theta_i, s\sin\theta_i - r\cos\theta_i)\, dr,$$

where $L_2^P(\mathbb{B}_1)$ is the P-wise Cartesian product of $L_2(\mathbb{B}_1)$, the Hilbert space of square integrable function supported on $\mathbb{B}_1 = [-1, 1]$. The traditional motion-free tomographic reconstruction problem is to:

Definition 1 (Motion-Free Tomography) *Estimate f from noisy measurements of $\mathcal{R}f$.*

Although this problem is well-studied, its formulation ignores one important physical consideration. In most data acquisition scenarios, these measurements are obtained *sequentially*, and f may change with time. When this change is significant, the motion-free tomographic model is no longer valid, and artifacts can occur in the reconstruction.

To handle the dynamic nature of f, we must redefine the problem. We now consider f to be a square integrable 3D density, where the third dimension is a time dimension. It is supported on

the cylinder $\mathbb{F}_3 = \mathbb{B}_2 \times [0,1]$, and is an element of the Hilbert space of such functions, denoted $L_2(\mathbb{F}_3)$. To convert the motion-free formulation to a more general one, we consider the following *time-sampled Radon Transform*: $\mathcal{R}_t : L_2(\mathbb{F}_3) \mapsto L_2^P(\mathbb{B}_1)$

$$\mathcal{R}_t f(s,i) = \int f(s\cos\theta_i + r\sin\theta_i, s\sin\theta_i - r\cos\theta_i, \tau_i)\, dr,$$

where $\tau_i \in [0,1]$. Note that this formulation allows for multiple projections to be acquired at the same time instant τ_i, simply by choosing $\tau_i = \tau_j$ for each pair of view angles i,j acquired simultaneously. The vector $\boldsymbol{\theta}$ of view angles and the vector $\boldsymbol{\tau}$ of time instants jointly characterize the imaging geometry. If, for example, the imaging device is an axial CT scanner, then the acquisition geometry is *uniform*, meaning that $\theta_k = k\Delta\theta, \tau_k = kT$. This acquisition geometry also occurs in the tomographic formulation of SAR.

The problem to be solved now is the following one:

Definition 2 (Time-sampled Tomography) *Estimate f from noisy measurements of $\mathcal{R}_t f$.*

B. Past Research

A diverse set of results have already been obtained on the time-sampled tomography problem. Broadly, these results can be divided into two parts. The first deal primarily with the parameters $\boldsymbol{\theta}$ and $\boldsymbol{\tau}$, and how they can be modified to make solution of the time-sampled problem feasible. The second set of results assume a fixed choice of $\boldsymbol{\theta}$ and $\boldsymbol{\tau}$ (usually a uniform geometry), and then study how different motion models affect solvability of the time-sampled tomography problem.

Modified Acquisition

Several researchers have looked at different means of choosing the geometry so as to best solve the time-sampled tomography problem. The definitive work in this area is to be found in [8, 9]. There, the authors examine the case in which f is spatially and temporally bandlimited. They construct, using an elegant lattice-packing argument, the optimal choices for $\boldsymbol{\theta}$ and $\boldsymbol{\tau}$ under these circumstances, as well as bounds on the minimum angular and temporal rates necessary to avoid aliasing in the reconstruction.

Similar results were obtained in [10], in which methods are described which adaptively choose "optimal" $\boldsymbol{\theta}$ based on f itself. There, the authors also provide efficient reconstruction methods

using the wavelet localization of the Radon transform [11]. The results provided are encouraging, demonstrating that the time-sampled tomography problem is at least theoretically solvable, provided the view angles can be placed correctly.

An entirely different approach in medical imaging is that of so-called *gated acquisition.* In this approach, it is generally assumed that f is periodic, and that some indicator of the phase within the period can be measured[1]. Specific approaches to gating are described in [12–14]. Generally, periodicity is exploited to reduce the time varying problem into a collection of static problems. Gating approaches suffer from both a requirement of cooperation on the part of the patient, and an assumption of "proper" biological operation to guarantee periodicity. Thus, imaging under unusual medical circumstances (such as cardiac arrythmia) can defeat gating based approaches. Furthermore, gated approaches are clearly useless for imaging nonperiodic phenomena. Tekalp offered artifact correction methods for periodic models without gating, in which the periodicity was extracted from the MRI data [15]. There, the out-of-slice motion was modeled as a modulation of the projection data. Haacke et al. have proposed Projection Onto Convex Sets (POCS) based reconstruction methods for periodic motion models, when the motion is partially known [16].

The general difficulty of these approaches in general, is that they require some modification of the method by which data is acquired. In many cases of practical interest, such as SAR, NDE, and Spiral CT, this is impractical or impossible. Although some technologies such as MRI or the Electron Beam Tomography [17] scanner permit great flexibility in the acquisition geometry, in the majority of applications, the geometry is fixed by physical considerations. For example, in spotlight-mode SAR the view angles are determined by the relative positions of the antenna and the target patch. Thus, physical constraints on the motion of the antenna (which might, for example, be attached to an aircraft) limit the flexibility of the acquisition sequence. Likewise, in CT scanners, the radiation source is typically mounted on a high speed rotating ring. The rotational motion restricts the acquisition geometry to be uniform [2].

Model based approaches

Other results on time-sampled tomographic imaging generally fix the imaging geometry, and

[1]Examples of indicators include ECG signals for cardiac imaging and chest displacement for imaging of the lungs.
[2]It is possible in theory to change the angular sampling pattern of a CT scanner by turning the x-ray source on and off at the appropriate times.

then study how different models for the time evolution of f affect solvability. Different researchers have adopted different models, generally motivated by physical concerns. For respiratory compensation in tomographic imaging, global and local affine models have been studied [18, 19]. For the global affine model, the function $f(\boldsymbol{x}, t)$ is related to some unknown static function $f_0(\boldsymbol{x})$ by

$$f(\boldsymbol{x}, \tau_i) = f_0(\boldsymbol{D}_i \boldsymbol{x} + \boldsymbol{b}_i, t)$$

where $\boldsymbol{D}_i \in \mathbb{R}^{2\times 2}$ are *known* diagonal matrices, and $\boldsymbol{b}_i \in \mathbb{R}^2$ are *known* shifts[3]. The local affine model is similar, except that the affine model is applied locally.

Some simple translational motion models have been studied in the context of different geometries, including 2D MRI[4] [20–22], SAR [23], and helical-CT [24]. These model-based approaches have been fairly successful, provided that the models adequately describe the motion.

Another class of models have been proposed by Liang et al [25–27]. Here, *a priori* knowledge of the spatial distribution of f for each sample instant is incorporated in the form of a specially chosen set of basis functions. A separable model of the form

$$f(\boldsymbol{x}, t) = \sum_n c_n(t)\phi_n(\boldsymbol{x})$$

is then used to model the temporal evolution of f. Assuming that the basis functions $\phi_n(\boldsymbol{x})$ can be chosen appropriately, these models can permit the study of "compartmentalized" dynamic changes, such as may occur in spectroscopy or in contrast studies. The coefficients $c_n(t)$ are uniquely determined by the data, provided that there are a sufficiently small number of degrees of freedom in the model [25].

In general, motion-model-based approaches permit the incorporation of specific prior information into the behavior of f. This can, of course, also be their Achilles heal, particularly if the resulting reconstruction technique is highly sensitive to deviations from the model.

[3]The diagonal matrices and shifts were obtained through external measurements, similar to gating, although periodicity was not assumed.

[4]Although the cited works were done using the rectilinear sampling strategy, and not the projection mode of acquisition, which is tomographic.

II. Fast Algorithms

A. Problem Formulation

In the realm of motion-free tomography, there are difficulties that arise in practice in the routine solution of the tomographic reconstruction problem. One important problem is the high computational requirement that real-time tomographic reconstruction places on existing computers. As medical X-ray CT scanners become more sophisticated, the reconstruction process becomes a major bottleneck in the processing of data for visualization [4]. Hence, there is a growing need for fast reconstruction algorithms that can produce diagnostic quality reconstructions in a small fraction of the amount of time currently needed.

B. Review

The traditional approach to accelerating the reconstruction process is to employ special purpose hardware, in the form of pipelined and hardwired architectures that perform the backprojection operation at high speed [28–30]. Parallel computing is another method used to speed up backprojection. These traditional approaches suffer from two major problems. First, the special purpose hardware quickly grows obsolete, as general purpose microprocessors become increasingly faster. Second, they only offer $O(1)$ (i.e. constant factor) speedups over a standard implementation on a serial processor.

Recently, a number of fast reconstruction algorithms have been proposed [31–33]. Each of these fast algorithms offers $O(N/\log N)$ speedup over direct implementation, where N is the linear size of the reconstructed image in pixels. The advantages of such algorithms over direct implementation for large N are obvious. With current sub-millimeter resolution CT scanners, $N = 10^3$ and larger images are becoming common place. Fast algorithms promise one to two orders of magnitude speedup for such images. And yet, the use of fast algorithms is not common. The main reason appears to be that each of these fast algorithms suffers from some fundamental limitation that leads to overall poor performance.

The oldest fast algorithms (and perhaps the most obvious) are known as Fourier Reconstruction Algorithms [34–38] (FRA). Based on the Fourier Slice Theorem, these algorithms interpolate the tomographic data from the polar on which they are acquired to a rectangular grid, both in the Fourier domain (see Figure 2.1. An inverse 2D-FFT can then be used to recover the cross-sectional

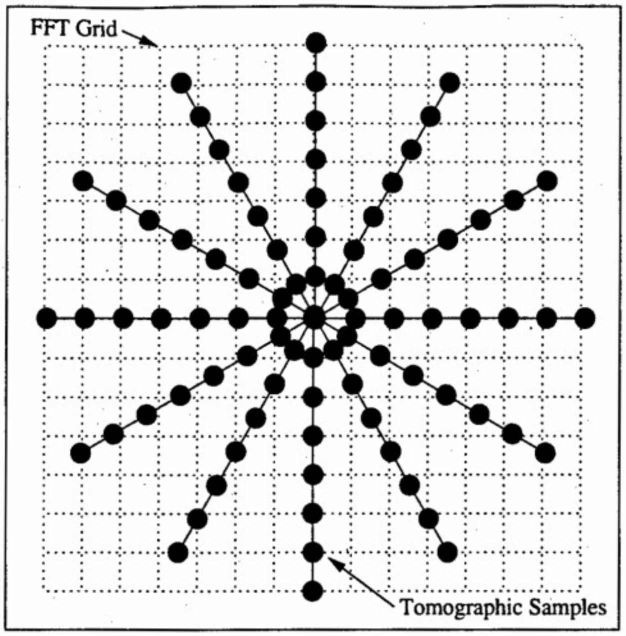

Figure 2.1: FRAs interpolate the Fourier data from the polar grid of the tomographic measurements, to the Cartesian, FFT grid.

density. In theory, this yields an $O(N^2 \log N)$ algorithm for reconstruction. But the reports of the performance of the FRA have been disappointing at best [39, 40]. In general, the precision necessary in the interpolation step results in an algorithm that, for reasonable sized images, is only slightly faster than the straightforward reconstruction algorithm, which requires $O(N^3)$ operations [41]. The latest incarnation of the FRA is known as gridding [31, 42], and includes all other FRA's as special cases. Gridding provides good reconstructions, but only provides small speedups for reasonable image sizes.

Next in line are the linogram reconstruction algorithms [32]. These algorithms are based on the assumption that the view angles lie at certain special angles, so that when the projections are cleverly resampled, the reconstruction can be performed using a series of 1D inverse FFTs. The requirements on the angles (and the radial resampling of the projections necessary in practice) make the resulting algorithms unattractive from a practical standpoint.

The third type of fast reconstruction algorithm is a multiscale approach [33]. In this algorithm, a sequence of nonuniform sample "grids" are used to merge the projection data in progressive steps, until the samples represent the reconstruction itself. Unfortunately, this algorithm generates blurred reconstructions, which must then be corrected using deconvolution techniques. As this

algorithm is fairly new, less is available as to its performance characteristics.

In addition to fast algorithms for reconstruction, there is also a need for fast algorithms to perform a variety of tomography-related computations. All of the methods described so far can be used for fast backprojection. Also of use are fast *reprojection* methods, in which projection-data are synthesized from an image. When fast reprojection and backprojection algorithms are combined, they can form the basis of fast and powerful iterative reconstruction techniques. In [43], just such an iterative technique is proposed. There, the reprojection and backprojection operations are combined into a single operation that can be computed efficiently using 2D FFT's. The resulting technique is both fast and powerful. Unfortunately, only operations on the measurement data that are radially shift invariant can be inserted between the reprojection and backprojection steps without destroying the convolutional nature of the algorithm. This limitation is rather restrictive, and significantly reduces the usefulness of the results in [43].

Recently, a fast reprojection algorithm was proposed that appears to suffer few of the problems that plague the other fast algorithms [7]. This fast algorithm is based on a fundamentally new decomposition of the Radon transform. It is capable of computing $O(N)$ projections from an $N \times N$ image in $O(N^2 \log_2 N)$ time, with small distortion. Our work on fast algorithms in tomography are based on the decomposition described in [7].

Chapter 3

Work Completed

I. Rigid Body Motion

Our work on tomography and motion has focused primarily on the use of specific kinds of parametric motion models to solve the time-sampled tomography problem with a fixed acquisition geometry. A surprising result that was first proposed simultaneously by Goncharov and Salzman, is that if f is defined by

$$f(\boldsymbol{x}, \tau_i) = f_0(\boldsymbol{Q}_i\boldsymbol{x} + \boldsymbol{b}_i)$$

where $\boldsymbol{Q}_i$ is a unitary matrix in $\mathbb{R}^{2\times 2}$, and $\boldsymbol{b}_i$ is sufficiently small so that $f(\boldsymbol{x}, \tau_i)$ is supported on $\mathbb{B}_2$, then the parameters $\boldsymbol{Q}_i$ and $\boldsymbol{b}_i$ that describe the "motion" at each sample instant can be determined from the projection data alone [44, 45]. Both Goncharov and Salzman were primarily interested in these results for the purposes of solving the alignment problem in Cryo-Electron Microscopy. This problem, illustrated in Figure 3.1, is to determine the orientations of a number of identical particles embedded in a solid, from a single projection, or very small number of projections. As discussed in [45], the problems are substantially different in 2D and 3D (as are many aspects of tomography), and the problem is nearly trivial in 3D.

A. Uniqueness

Although the idea that tomography in the presence of rigid body motion was suggested by Salzman and Goncharov, they failed to demonstrate rigorously that the parameters of the rigid body motion model were indeed uniquely recoverable. A guarantee of uniqueness may be critical in

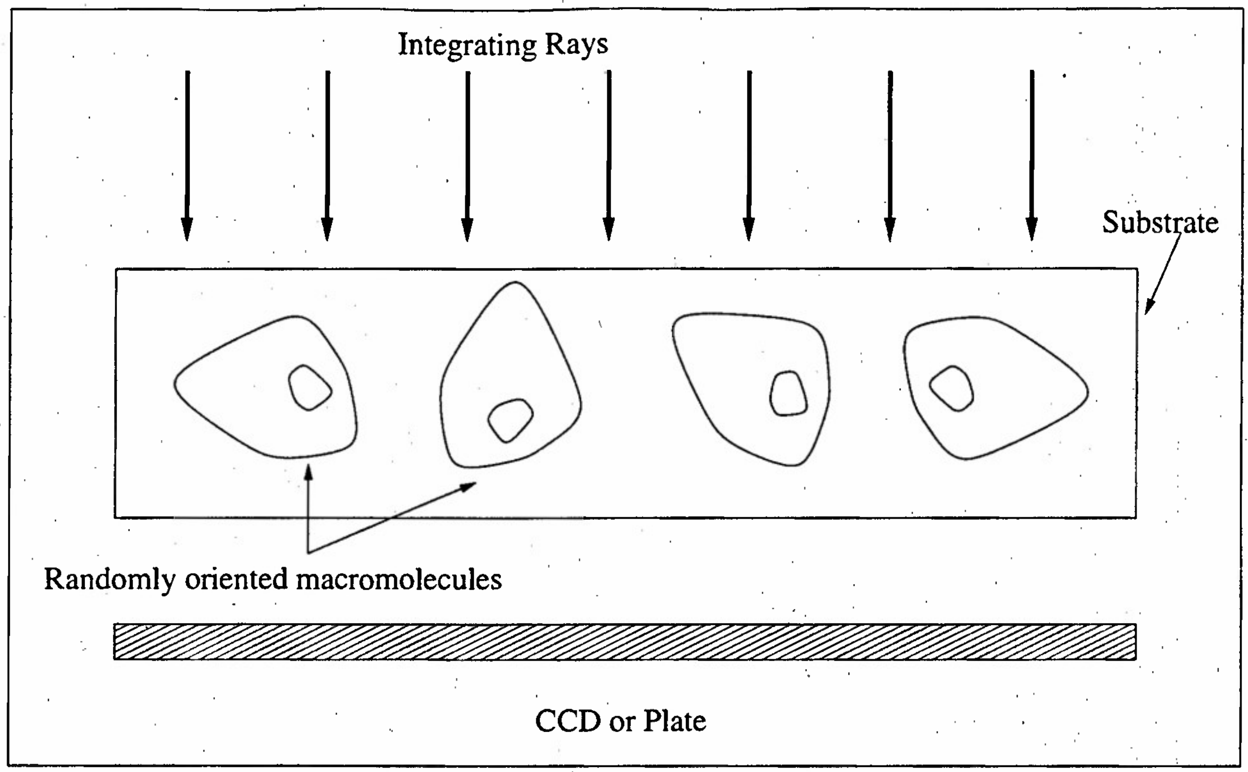

Figure 3.1: How the unknown view angle problem arises in Cryo-EM imaging of macromolecules.

sensitive applications, such as medical imaging, where nonunique solution may lead to misdiagnosis.

A major accomplishment of our research under this fellowship was to demonstrate carefully and rigorously that the motion parameters are uniquely determined by the projection data, and to exactly determine the conditions that must be satisfied for these uniqueness conditions to hold. In [46, 47], we rigorously demonstrated the following fact:

Result 1 *For almost all f_0, and almost all $\boldsymbol{\theta}$ with $P > 24$, $\boldsymbol{Q}_i$ and $\boldsymbol{b}_i$ are uniquely determined by $\mathcal{R}_t f$.*

Furthermore, our techniques laid the foundation for future work, in the form of more complex motion models, and a mechanism for determining conditions for unique recovery of motion parameters from projection data.

B. Stability

Having established that the motion model parameters were uniquely determined from the projection data, the next step was to study the effect of noise in the projection data on the solution

for the motion parameters. In [48], we proved the following fact:

Result 2 *For almost all f_0, and almost all $\boldsymbol{\theta}$ with $P > 24$, $\boldsymbol{Q}_i$ and $\boldsymbol{b}_i$ are uniquely determined by $\mathcal{R}_t f$, and furthermore, the problem of determining $\boldsymbol{Q}_i$ and $\boldsymbol{b}_i$ from $\mathcal{R}_t f$ in the presence of noise is stochastically stable, in the sense that the Cramér-Rao bounds are finite.*

We also elucidated the connection between the Cramér-Rao bounds and a deterministic notion of stability in [49]. With the aid of this analysis, we obtained the following additional result:

Result 3 *For almost all f_0, and almost all $\boldsymbol{\theta}$ with $P > 24$, $\boldsymbol{Q}_i$ and $\boldsymbol{b}_i$ are uniquely determined by $\mathcal{R}_t f$, and furthermore, the problem of determining $\boldsymbol{Q}_i$ and $\boldsymbol{b}_i$ from $\mathcal{R}_t f$ in the presence of noise is deterministically stable, in the sense that the least squares estimate depends smoothly on the data for any sufficiently small perturbation of the data.*

Showing that the bounds were finite was a first step. We also obtained the following (numerical) result:

Result 4 *The rotational and translational components of the rigid motion can (according to the Cramér-Rao bounds) be estimated to within a fraction of a degree and a small percentage, respectively, at an SNR of 30dB.*

These results suggest that the problem of determining $\boldsymbol{Q}_i$ and $\boldsymbol{b}_i$ from $\mathcal{R}_t f$ is feasible, provided the noise level is sufficiently small.

Our final result on this problem (also reported in [48]) was a simple algorithm to estimate $\boldsymbol{Q}_i$ and $\boldsymbol{b}_i$:

Result 5 *A simple and efficient algorithm exists that can estimate $\boldsymbol{Q}_i$ and $\boldsymbol{b}_i$, and that achieves the Cramér-Rao bound for SNR $\geq$ 20dB on a computer phantom.*

It is important to point out that while none of the previous results made assumptions as to the nature of $\boldsymbol{Q}_i$ and $\boldsymbol{b}_i$, the algorithm we proposed made some assumptions. Specifically, the proposed algorithm assumes that there exists a permutation $\boldsymbol{\pi}$ of the projections such that $\boldsymbol{Q}_{\pi(i)} \approx \boldsymbol{Q}_{\pi(i+1)}$ and $\boldsymbol{b}_{\pi(i)} \approx \boldsymbol{b}_{\pi(i+1)}$. Thus, under some (unknown) reordering of the samples, the motion described by $\boldsymbol{Q}$ and $\boldsymbol{b}$ was small between samples. The algorithm then searches for the correct permutation of the projections. In [48], we demonstrated that this strategy works well if a large number of projections are available without significant "gaps" between them.

II. Fast Algorithms for Tomography

A. Fast Backprojection

As we mentioned in Chapter 2, the introduction in [7] of a fundamentally new decomposition of the Radon transform has lead to a new class of fast algorithms for reprojection, i.e. the computation of projections from a discrete image. Unlike direct computational methods, which require $O(N^3)$ operations to compute N projections from and $N \times N$ image, the new fast algorithm of [7] promises the same results, but in $O(N^2 \log_2 N)$ operations. For large image sizes, the potential image speedup is enormous.

The operation of reprojection is useful in tomographic reconstruction, particularly in the context of iterative algorithms. But the operation of *backprojection* is even more prevalent. Given a collection of P projections $g_c(r,p)$ for $r \in \mathbb{R}$, and $p \in \{1, \dots, P\}$, backprojection requires computing

$$f(i,j) = \sum_{p=1}^{P} g_c(i \cos \theta_p + j \sin \theta_p, p) \tag{2.1}$$

where θ_p is the p-th view angle. A straightforward discretization of (2.1) leads to an $O(N^3)$ algorithm if $P = O(N)$.

However. we have been able to use the same type of decomposition described in [7] to generate a fast algorithm to perform this same computation in $O(N^2 \log_2 N)$ operations [50]:

Result 6 *A simple, fast and controllable algorithm to compute the backprojection of P projections on an $N \times N$ pixel grid in $O(NP \log_2 N)$ time.*

Experimental results demonstrate that the new algorithm achieves the promised $N/\log_2 N$ speedup for large images:

Result 7 *The resulting fast algorithm can compute backprojections for 1024×1024 pixel images over 90 times faster than direct methods, and faster, and with lower distortion than any other fast reconstruction method available at the time.*

III. Fundamental Results in Signal Processing

As a byproduct of our primary research goals, we have developed some fundamental results in signal processing.

A. Global Bounds on Estimation

In many signal and image processing problems, one must estimate a vector parameter $\boldsymbol{\theta}$ given a noisy message $g(t, \boldsymbol{\theta})$ that is parameterized by $\boldsymbol{\theta}$. Furthermore, in many situations of practical interest, the g depends on some components of $\boldsymbol{\theta}$ in a periodic manner. Examples of problems of this kind include phase estimation from a periodic pulse train. In this example, the message g depends on the phase in a periodic manner. Image registration is another example. Here, the vector $\boldsymbol{\theta}$ may represent the rotation and shift necessary to co-align two images. In that case, the dependency of g on the rotational parameter is periodic. Another example is orientation estimation. In this example, the $g(t, \boldsymbol{\theta})$ represents a signal, like a radar or sonar return of an object in unknown position. The dependence of g on the positional parameters corresponding to roll, pitch and yaw, is periodic.

In all of these cases, some of the parameters affect the message in a periodic manner, and the other parameters affect the message in a nonperiodic manner. For parameters of the first kind, the correct choice for measuring distortion is to use a periodic distortion measure. For example if θ is a phase, then a good distortion metric might be

$$D(\theta - \hat{\theta}) = 1 - \cos(\theta - \hat{\theta}). \tag{3.1}$$

Once a distortion measure is chosen, a problem of interest is to find fundamental, global lower bounds on the minimum distortion achievable by an estimator for θ. Such bounds serve as benchmarks for evaluating algorithms that approximate estimators. If an algorithm achieves the global lower bound, then it cannot be improved upon in terms of distortion performance.

Unfortunately, past research in this area is incapable of dealing with periodic distortion functions such as (3.1). However, while pursuing our primary research goals, we developed the following [51]:

Result 8 *A global lower bound on estimation error for vector parameters where some of the distortion functions are periodic.*

B. Stability of Nonlinear Least Squares Problems

Another result we have established while pursuing our primary goals is in regards to the stability of nonlinear least squares problems. A fair number of problems in signal and image processing can

be reduced to the solution of a nonlinear least squares problem:

$$\hat{\boldsymbol{y}} = \arg\min_{\boldsymbol{y}} \|\boldsymbol{f}(\boldsymbol{y}) - \boldsymbol{b}\|_2 \tag{3.2}$$

where $\boldsymbol{y} \in \mathbb{R}^m$, $\boldsymbol{b} \in \mathbb{R}^n$, and $\boldsymbol{f} : \mathbb{R}^m \mapsto \mathbb{R}n$ is a nonlinear, smooth (i.e. twice continuously differentiable) function.

Often, when considering stability of (3.2), one is forced to consider the problem in a stochastic setting. There, one assumes that the measurements are Gaussian distributed with known variance, $\boldsymbol{b} \sim N(\boldsymbol{f}(\boldsymbol{y}), \sigma^2 \boldsymbol{I})$, where $\boldsymbol{I}$ is the $n \times n$ identity matrix. With this assumption, it then becomes possible to compute Cramér-Rao bounds on variance of unbiased estimators of $\boldsymbol{y}$ given $\boldsymbol{b}$. However, the resulting bounds are valid only if the measurements are Gaussian. In the presence of small, systematic errors, the measurement errors may be better modeled as deterministic, instead of stochastic. In [49], we proved the following result:

Result 9 *There are a set of conditions which guarantee stability of* (3.2) *in both the traditional stochastic sense, and in the deterministic sense. Hence, if these conditions are satisfied, then the Cramér-Rao lower bound on the variance of unbiased estimators is finite, and the estimate $\hat{\boldsymbol{y}}$ varies smoothly with any (deterministic) perturbation of the data $\boldsymbol{b}$, provided the perturbation is sufficiently small.*

We have applied this result to two classic problems in signal processing with the following new results:

Result 10 *The sinusoid retrieval problem is deterministically stable.*

Result 11 *The multichannel, blind deconvolution problem is deterministically stable.*

Chapter 4

Conclusions

In this report, we have outlined the research we have performed while sponsored by the JSEP fellowship. We have made significant and fundamental contributions to the areas of tomography in the presence of motion, fast algorithms for tomographic reconstruction, and stability and estimation problems in signal processing.

Publications

In this appendix, we list separately, the publications written during the JSEP fellowship. These publications constitute a thorough, technically accurate documentation of the work accomplished. In keeping with the instructions for preparation of this document, none of these publications are attached.

1. S. Basu and Y. Bresler, "Uniqueness of Tomography with Unknown View Angles." To appear in *IEEE Transactions on Image Processing.*

2. S. K. Basu, "Tomography With Unknown View Angles," Master's thesis, University of Illinois at Urbana-Champaign, 1998.

3. S. Basu and Y. Bresler, "Feasibility of Tomography with Unknown View Angles." To appear in *IEEE Transactions on Image Processing.*

4. S. Basu and Y. Bresler, "The Stability of Nonlinear Least Squares Problems and the Cramér-Rao Bound." Accepted to the IEEE Transactions on Signal Processing.

5. S. Basu and Y. Bresler, "Fast Filtered Hierarchical Backprojection Algorithm for Tomography." Submitted to *IEEE Trans. Imag. Proc.*

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7. S. Basu and Y. Bresler, "Further Results on Tomography with Unknown View Angles," in *Proc. IEEE Int. Conf. Image Proc.*, (Chicago, IL), 1998.

8. S. Basu and Y. Bresler, "Tomography with Unknown View Angles," in *Proc. IEEE Int. Conf. Acoust. Speech and Sig. Proc.*, vol. 4, (Munich, Germany), pp. 2849–2852, Apr. 1997.

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PREV DEGREE: UNIVERSITY OF DELAWARE
BEE 1995

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CRSE	NUMB	CRSE DESCRIPTION	CREDIT	GRD NTE
Graduate				
		ELECTRICAL ENGINEERING		
FALL SEMESTER 1995				
ECE	400	SEMINAR	0.00¤	S
ECE	415	CONT SYST THRY & DES	1.00¤	A
ECE	434	RANDOM PROCESSES	1.00¤	A
ECE	451	DIGITAL SIGNAL PROC	1.00¤	A
ECE	499	THESIS RESEARCH	(0.50¤)	DF
SPRING SEMESTER 1996				
ECE	361	DIGITAL COMM	0.75¤	A
ECE	373	FUND ENGR ACOUSTICS	1.00¤	A
ECE	390	INTRO OPTIMIZATION	1.00¤	A
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ECE	499	THESIS RESEARCH	(0.25¤)	DF
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ECE	499	THESIS RESEARCH	(2.00¤)	DF
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DEGREE: M.S. (ELECTRICAL ENGINEERING)
DATE: January 15, 1998

CRSE	NUMB	CRSE DESCRIPTION	CREDIT	GRD NTE

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UIUC GRADUATE GPA=4.000 BASED ON 14.75 UNITS
TOTAL GRADUATE UNITS EARNED= 15.25

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TRANSCRIPT EXPLANATION

I. CREDIT

A. University Credit

Courses shown on the transcript reflect all work attempted while a student at the Urbana-Champaign campus.

1. Undergraduate and professional credit is recorded in semester hours. Each semester hour represents one fifty-minute period of classroom work each week for the duration of one semester (two periods of classroom work per week during an eight-week summer session) or the equivalent in laboratory, field work or approved independent study.

2. Graduate credit is recorded in semester units and is identified by a lozenge (¤) immediately following the credit entry. Each semester unit of graduate work is considered equivalent in quantity of classroom work to four semester hours.

B. Transfer Credit

1. Undergraduate transfer credit shown on the transcript is that credit accepted for admission purposes by the University of Illinois. It is recorded in semester hours.

2. Graduate transfer credit shown on the transcript is that credit accepted for degree purposes by the Graduate College. It is recorded in semester units.

C. Inter-campus Graduate Program Credit

When only one of its campuses has degree-granting authority in a particular area, the University offers an inter-campus degree program. This program is based on cooperative agreement between any two or more departments on different campuses of the University, thereby permitting a candidate to earn a degree at the master's or doctoral level.

Credit earned in an inter-campus graduate program at the Chicago campus is recorded on the transcript as quarter-hour credit prior to Fall Semester 1991. In this case, the presence of the lozenge (¤) designates graduate course work, not semester units. Credit is converted to units for the purposes of including it in the UIUC cumulative graduate grade-point average (GPA) and graduate units earned.

D. Religious Foundation Course Credit

Courses offered by the religious foundations located in Urbana-Champaign are accepted for credit by the University. No grades are recorded on the transcript for religious foundation courses and the credit is not included in the cumulative GPA. The symbol "**" will appear in the grade column on the transcript.

E. Extramural Course Credit

Extramural courses are offered for credit at off-campus locations and are listed separately on the transcript. Credit in extramural courses may be in units or hours, and it is included in the UIUC cumulative GPA and cumulative hours/units earned. Beginning Fall Semester 1983, 300 and 400 level extramural courses bearing unit credit are approved by the Graduate College for inclusion in a student's degree program as credit taken in residence.

Prior to Fall Semester 1983, there were two types of extramural courses. Extension (E) courses were conducted by University faculty members and were directed primarily to a continuing education audience. Graduate Residence (R) courses were conducted by University faculty members and were approved for graduate resident credit by the Graduate College. The (E) or (R) designation immediately preceded the course number. For spring semester and Summer Session 1983, Graduate Residence (R) courses were identified only by location of instruction.

F. Correspondence Course Credit

Correspondence courses conducted by University faculty members and administered by the Division of University Extension are accepted for credit. Correspondence courses and the dates the student was enrolled are listed separately from traditional courses on the transcript under the CORRESPONDENCE heading. An "X" immediately precedes the course number. Credit in correspondence courses is included in the UIUC cumulative GPA and cumulative hours earned.

II. GRADE EXPLANATION

A. Grades Included in Calculation of Grade-Point Averages

1. ALL COLLEGES[1]:

Grade	Grade Points	Grade	Grade Points
A+	=4.00	C+	=2.33
A Excellent	=4.00	C Fair	=2.00
A-	=3.67	C-	=1.67
B+	=3.33	D+	=1.33
B Good	=3.00	D Poor	=1.00
B-	=2.67	D-	=0.67
F Failure (including courses dropped for academic irregularities).	=0.00		
**F F by rule. Grade of "F" on the letter scale has replaced "EX" because of student's failure to comply with specified time limitation.			

2. COLLEGE OF LAW ONLY:
   a. Prior to Fall 1992, grades of B+ (3.5) and C+ (2.5) were used. Beginning with the Fall Semester 1992 the law grades are the same as those shown in "1".

B. Grades Not Included in Calculation of Grade-Point Averages

CR	Spring Semester 1975 to present. Used only in courses taken under the credit/no credit grading option. A minimum grade of "C" is required for credit.[2]
DF	Grade temporarily deferred. Used only in graduate and undergraduate thesis and honors courses, and in a limited number of other courses that extend over more than one semester.
EX	Approved extension of time to complete the final examination or other requirements of the course.
FF	Fail[3]
IP	Course in progress
MISS	Missing Grade
NC	No credit[2]
O	Outstanding (School of Basic Medical Sciences only).
P	Pass[3]
PASS	Used for test-based credit. A minimum grade of "C-" is required.
S	Satisfactory[4]
SA	Graduate transfer credit.
U	Unsatisfactory[4]
W	Approved withdrawal without credit.
**	Used for courses in which no grades are recorded such as transfer credit, religious foundation courses, etc.

---

[1] Prior to the Fall Semester 1996, plus and minus grades were not awarded.

[2] CR/NC: Credit/no credit (Prior to Spring Semester 1975, CR was assigned for satisfactory completion of course work taken through the Study Abroad Program with a grade of "D" or better).

[3] P/F: Spring Semester 1968 through Fall Semester 1974. Used only in courses taken under the pass/fail grading option. A minimum grade of "D" was required for passing.

[4] S/U: Used only as final grades in graduate thesis research courses, in graduate and undergraduate courses given for zero credit, and in certain other specified courses.

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